# Discrete Optimization - Assignment 2

## Theoretical part

First, we start by clarify the notation. We are given element and sets where . Further we are given a cost function defined for each . The ILP problem is:

Solving the LP-relaxation yields a potentially fractional solution, . We then let be the sets that are picked in the fractional solution. That is, . Note that for due the constrains the problem; if than one could reduce the primal variable and still satisfy all constrains for edges that is covered by

Going through the random rounding algorithm in section 14.2 yields a solution where repetitions is chosen such that:

It then follows that:

Varzirani then use the union bound to compute the upper bound for the event that is not a cover by:

Now, take a subset of such that . It then follows from the union bound that:

If the event occur then does not cover at least half of the elements. We can only make two of such disjoint sets. Let and . Then from the union bound it follows that:

This is a weak bound for the probability that at least half of the elements is not covered given that it is the same bound as just one element not being covered.

Next, we turn to the probability that the cost exceeds . The integrality gap of LP relaxation is as argued in Vazirani on page 111:

The corresponds between the logarithm and the harmonic series is:

Thus, the complementary probability of interest is:

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Hence, the upper bound for the non-covered elements have an upper bound given by a binomial distribution with elements and a probability . Let . Then: