# Discrete Optimization - Assignment 2

## Theoretical part

First, we start by clarify the notation. We are given element and sets where . Further we are given a cost function defined for each . The ILP problem is:

Solving the LP-relaxation yields a potentially fractional solution, . We then let be the sets that are picked in the fractional solution. That is, . Note that for due the constrains the problem; if than one could reduce the primal variable and still satisfy all constrains for edges that is covered by

Going through the random rounding algorithm in section 14.2 yields a solution where repetitions is chosen such that:

It then follows that:

Varzirani then use the union bound to compute the upper bound for the event that is not a cover by:

Now, take a subset of such that . In the same spirit, using Fréchet inequalities yield:

If the event occur then does not cover at least half of the elements. There are distinct set of this size. As fare as we know, we cannot assume independence. Thus, we are left with by the union bound of these event we get some thing far greater than . However, there must occur at least one event for half the sets not to be covered. Thus:

This is a weak bound for the probability that at least half of the elements is not covered given that it is the same bound as just one element not being covered

Next, we turn to the probability that the cost exceeds . The integrality gap of LP relaxation is the harmonic series, , as argued in Vazirani on page 111:

Thus, the complementary probability of interest is:

I.e. best bet so fare is:

## Performance table

Below is a table of the result and running time. Running times are reported as [Algorithm running time] / [Algorithm running time + time for loading the instance]. All running times are computed when solely running the program with the particular algorithm to avoid problems with chacing distorting the comparison. Note that we compute a mean, minimum and maximum for random rounding over 200 samples. The reported running time is for the first solution:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Random rounding | | | | Rounding | | Primal-dual | | Cplex exact | | Clpex LP-relaxation | |
| Instance | (first calculation) Running time | Min | Max | Mean | Running time | Value | Running time | Value | Running time | Value | Running time | Value |
| scpa3 | 108/152 | 415 | 487 | 451.36 | 109/135 | 490 | 2.24/27 | 448 | 346/373 | 232 | 113/139 | 228.00 |
| scpc3 | 173/216 | 480 | 580 | 529.26 | 125/163 | 587 | 2.42/39 | 463 | 850/892 | 243 | 122/155 | 234.54 |
| scpnrf1 | 320/495 | 39 | 62 | 51.25 | 321/484 | 78 | 2.02/160 | 43 | 12,165/12,308 | 14 | 314/450 | 8.89 |
| scpnrg5 | 322/466 | 467 | 541 | 513.80 | 309/416 | 621 | 3.93/105 | 412 | - | - | 290/385 | 148.23 |